A Prime Investigation with 7, 11, and 13

ID: 13344

Time required
45 minutes

Activity Overview

In this activity, students will investigate the divisibility of 7, 11, and 13 and discover the divisibility characteristics of certain six-digit numbers. They will use the Integer Division feature of the TI-73 Explorer.

Topic: Numbers and Operations

- *Use factors, multiples, prime factorization, and relatively prime numbers to solve problems*

Teacher Preparation and Notes

- *Prior to beginning this activity, students should have learned the divisibility tests for the prime numbers 2, 3, and 5.*
- *Students should be familiar with the term remainder and what it represents.*
- *TI-Navigator is not required for this activity, but an extension is given for those teachers that would like to use it.*
- ***To download the student worksheet and TI-Navigator files, go to education.ti.com/exchange and enter "13344" in the quick search box.***

Associated Materials

- *MGAct04_Prime_worksheet_TI73.doc*
- *MGAct04_Prime_Nav_TI73.act*
- *MGAct04_Prime_LrnChk_TI73.edc*

Suggested Related Activities

To download the activity listed, go to education.ti.com/exchange and enter the number in the quick search box.

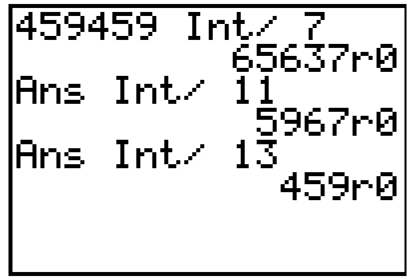
- *Prime Investigation of 7, 11, and 13 (TI-73 Explorer) — 4473*
- *Priming the Numbers (TI-73 Explorer) — 8452*
- *Prime Factorization, GCF, and LCM (TI-73 Explorer & TI-Navigator) — 10465*

Problem 1 – Divisible by 7, 11, and 13?

Students will explore divisibility by 7, 11, and 13 using the [INT÷] feature. The part after the **r** is the remainder, if one exists. It will display a 0, if one does not exist.

Questions 1-4

Have students work in small groups throughout the activity. Each student should pick a unique number to work with throughout the problem.



Once students have picked a six-digit number, have them enter this on a clear home screen. Then, press [2nd] [÷] [7] [ENTER]. As you can see, the answer is shown with the quotient and remainder.

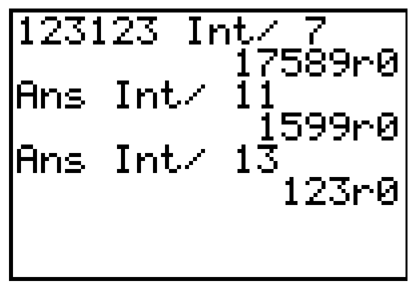
If student have not worked with the [INT÷] feature before, you may want to spend a little time with smaller numbers so they understand how the function works.

If students have correctly created their six-digit number, they should find that all three division steps results in no remainder.

Problem 2 – Justify Divisibility

Questions 7-9

Again, have students work in small groups. Each student should pick a unique number to work with throughout the problem.



Again, their six-digit number (*abc*,*abc*) should be divisible by 7, 11, and 13.

Questions 10-14

As students explore the different relationships, encourage conversation and discussion around the reasons behind what is happening.

Students should see that when they multiply their original 3-digit number (*abc*) by ($7 \times 11 \times 13$), that they end up with the original 6-digit number (*abc*,*abc*).

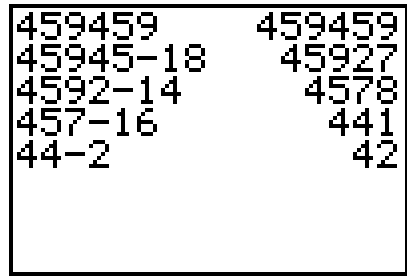
Problem 3 – Divisibility Tests

Questions 15-16

In case students do not remember the divisibility tests for 2, 3, and 5, they are given below. Direct conversation to help them recall these rules.

- A number is divisible by 2 if it is even or if the ones digit is 0, 2, 4, 6, or 8.
- A number is divisible by 3 if the sum of the digits are divisible by 3.
- A number is divisible by 5 if the ones digit is a 0 or a 5.

Make sure that students see that they start each new step with the number that was created in the previous step, not the original number from the previous step. This part of the process may initially be confusing to students.



<u>Questions 17-22</u>

Having students explore the 11 divisibility rule exposes them to additional number theory. This is a bit of trivia that is really not needed if a calculator is readily available.

The last question should spark lots of discussion among the students' groups. If they quickly come up with a rule for divisibility by 1001, you can have them revisit the rules for 2, 3, 5, 7, or 11 and see if they can find a counterexample.

Extension – TI-Navigator™

1. For Problem 1 you can have students load the six-digit numbers used by their group into a list for the class to share. Load the activity settings file ***MGAct04_Prime_Nav_TI73.act*** into Activity center. Start the activity when students are ready to submit the numbers used by their group. After all data points have been received from the students the data can be sent back out to all students. Stop the activity, click on **Configure** and click the button for "**Existing Activity List**". This will send the class set of data back to all students.
2. Use **Screen Capture** to monitor student activity throughout the lesson.
3. As an assessment, send ***MGAct04_Prime_LrnChk_TI73.edc*** to students as an evaluation. This LearningCheck™ file evaluates student understanding of divisibility rules.

Solutions – student worksheet

Problem 1

1. Answers will vary.
2. Yes, students' numbers should be divisible by 7.
3. Yes, students' numbers should be divisible by 11.
4. Yes, students' numbers should be divisible by 13. The new quotient is the original 3-digit number. No, the order of division will not change the outcome.
5. Everyone's number should be divisible by 7, 11, and 13.
6. If you multiply $7 \times 11 \times 13$ you get 1001 and if you take 1001 times any three-digit number you get a six-digit number where the original three-digit number repeats. Therefore, the original six-digit number would be divisible by 1001 or 7, 11, and 13.

Problem 2

7. Answers will vary.
8. Yes, students' numbers should all be divisible by 7, 11, and 13.
9. The six-digit number (*abc*,*abc*) should be divisible by 7, 11, and 13.
10. 1,001
11. The original six-digit number – *abc*,*abc*. It is the same.
13. $abc \times 1001 = abc \times (1000 + 1)$

$$= abc \times 1000 + abc \times 1$$

$$= abc{,}000 + abc$$

$$= abc{,}abc$$

14. Since $468{,}468 = 468 \times 1001$ and $1001 = 7 \times 11 \times 13$ then 468,468 is divisible by 7, 11, and 13.

Problem 3

15. A number is divisible by 2 if it is even or if the ones digit is 0, 2, 4, 6, or 8. A number is divisible by 3 if the sum of the digits is divisible by 3. A number is divisible by 5 if the ones digit is a 0 or a 5.
17. Yes, it should show that the original number is divisible by 7.
20. Yes, it should show that the original number is divisible by 11.
21. No. Since $8 + 2 + 4 = 14$ and $5 + 3 + 5 = 13$ and $14 - 13 = 1$ and 1 is NOT divisible by 11, then 852,345 is not divisible by 11.
22. Answers will vary. Check student's work. A six-digit number is divisible by 1001 if it is in the form *abc*,*abc*.

Name ________________________

Class ________________________

Problem 1 – Divisible by 7, 11, and 13?

To begin, pick any 3-digit number and repeat it to create a 6-digit number. (459,495 is used here as an example)

1. Write down your number. Make sure everyone in your group has a different number.

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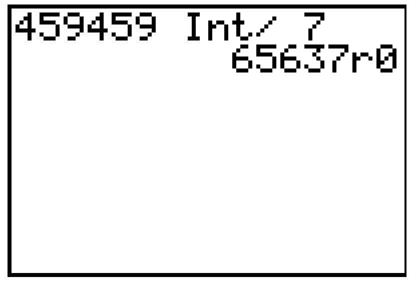

2. Enter your number on the Home screen and then divide by 7 to see if you get a remainder of 0. (Enter the number then press [2nd] [÷] [7] [ENTER].)

Is your number divisible by 7? ________________

3. Next, divide the previous quotient by 11. Press [2nd] [÷] [1][1] [ENTER].

Is your number divisible by 11? _______________

4. Next, divide the previous quotient by 13. Press [2nd] [÷] [1][3] [ENTER].

Is your number divisible by 13? _______________

What do you notice about the new quotient? ________________________________

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Would the order that you divided by 7, 11, or 13 affect your result? Try dividing your original number by these divisors in different orders. Write a sentence on your findings.

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5. Did everyone in your group pick a number that was divisible by 7, 11, and 13? ______

6. If so, discuss within your group why you think everyone's number was divisible by 7, 11, and 13. Write a sentence or two explaining why you think these special six digit numbers are divisible by 7, 11, and 13. ______________________________________

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Problem 2 – Justify Divisibility

Now you will justify the findings from Problem 1 on divisibility.

7. Create another six-digit number by picking a 3-digit number and repeating it. Write down your number. Make sure everyone in your group has a different number.

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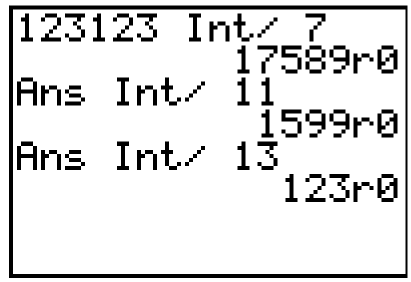
8. Test your number for divisibility by 7, 11, and 13 like you did in Steps 2-4 above. (Enter your number then press [2nd] [÷] [7] [ENTER] [2nd] [÷] [1][1] [ENTER].[2nd] [÷] [1][3] [ENTER].)

Is your number divisible by all three? __________



9. Write a sentence explaining your findings from Step 8.

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10. Multiply $7 \times 11 \times 13$. What is the product? ______________

Divide your number in Step 7 by this product. Answer: ______________

11. Multiply your 3-digit number in Step 7 by this product. Answer: ______________

Describe the relationship between this product and the special six-digit numbers you created in Step 1 and Step 8. ____

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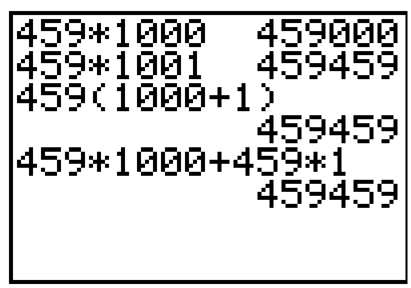
12. Examine the screen shot at the right. Multiplying by 1000 is easy to do mentally because you just move the decimal point 3 places to the right.

Multiplying by 1001 is also easy when you think of 1001 as 1000 + 1 and then multiply 459 x 1000 and add 459 x 1. This is known as the **distributive property**.



13. Rewrite your three-digit number as the product of the number and 1001. Then write it using the distributive property as shown in Step 12. ______________________

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14. Explain how you know 468,468 is divisible by 7, 11, and 13. _________________

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Problem 3 – Divisibility Tests

Next you'll look at divisibility tests for other prime numbers.

15. Write the divisibility tests for 2, 3, and 5.________________________________

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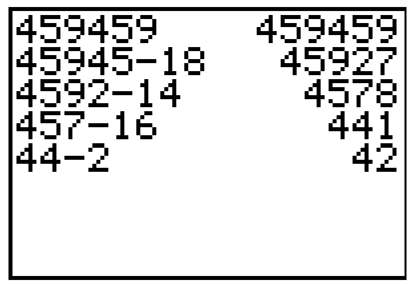
16. There's also a divisibility test to tell if a number is divisible by 7. Here is how it works:

- Take all but the last digit (the ones digit) and form a number.
- Subtract twice the ones digit from the number you formed. Now you have a new number.
- Again, take all the digits but the ones digit and form a new number.
- Subtract twice the ones digit from this number.
- Continue this process until you are able to recognize whether the number is divisible by 7. See the screen to the right.

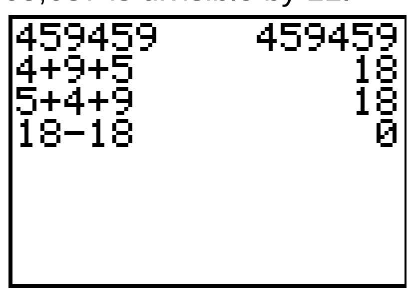


17. Apply this divisibility rule to your original number in Step 1. Did the test show that your number is divisible by7? _______________

18. There is a divisibility test for 11. For example, **6**5,**6**3**7** is divisible by 11. To use the test, sum every other digit, and then take the difference in the two sums.

- The sum of the digits in bold is 6 + 6 + 7 = 19.
- The sum of the underlined digits is 5 + 3 = 8.
- Now take the positive difference in the two sums and see if the result is divisible by 11. Since 19 – 8 = 11 and 11 is divisible by 11, then 65,637 is divisible by 11.

19. In the example at the right, since 18 – 18 = 0, and 0 is divisible by 11, the original number is divisible by 11.



20. Show how to use the divisibility test for 11 on your original number in Step 1? Does the divisibility test show what you previously found? _________

21. Use the divisibility test for 11 to show if 852,345 is or is not divisible by 11.

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22. Number theorists have developed divisibility rules or test for many different numbers. Can you write a divisibility test for a six-digit number to be divisible by 1001?

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